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Dual Field Theory of Strong Interactions

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A dual field theory of strong interactions is derived from a Lagrangian of the Yang-Mills and Higgs fields. The existence of a magnetic monopole of mass 2397 MeV and Dirac charge $g=(137/2)e$ is incorporated into the theory. Unification of the strong, weak, and electromagnetic forces is shown to converge at the mass of the intermediate vector boson $W^{\pm}$. The coupling constants of the strong and weak interactions are derived in terms of the fine-structure constant $\alpha=1/137$.

1. INTRODUCTION

In a recent paper (Akers, 1986), evidence was presented for the existence of a magnetic monopole of mass 2397 MeV and Dirac charge $g=(137/2)e$. From the attempt to reconcile its existence with grand unification theories (GUT), the low-mass magnetic monopole was incorporated into a dual field theory of the strong interactions called magnetostrong theory. The idea that magnetic charge accounts for the existence of strong forces in nature has its beginnings with Schwinger's magnetic model of matter (Schwinger, 1969) and later with Barut's model of hadrons (Barut, 1971a,b).

The present investigation is inspired by the results of magnetostrong theory (Akers, 1986), which yields calculations of the strong coupling constant α_s in excellent agreement with experimental data. In this paper it is suggested that a gauge theory of $SU(3)$ magnetic monopoles allows for the existence of scalar mesons as exchange particles in strong interactions and for the appearance of vector bosons in weak interactions.

In Section 2 the exchange particles in strong interactions are argued to be $\pi^{\pm}$ mesons. The electron is treated as an isovector particle in isospin

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space. In Section 3 the magnetic monopole is treated as an isovector particle, and we derive the same results for the $\pi^{\pm}$ meson mass as found in Section 2. In Section 4 we derive the fundamental coupling constants of the strong and weak interactions in terms of the fine-structure constant $\alpha = 1/137$. Concluding remarks are made in Section 5.

2. ELECTRONS AS ISOVECTOR PARTICLES IN ISOSPIN SPACE

In this section, it will be argued that $\pi^{\pm}$ mesons are the exchange particles of the strong interaction. Electrons will be treated as isovector particles ($I=1$) in order to derive the mass of the pions. This assignment of isotopic spin to the electron is based upon the conservation of electric charge and the conservation of isotopic spin in strong interactions. As shown in Table I, the quantum numbers Q/e and I_3 are related through the Gell-Mann-Nishijima formula $Q/e = I_3 + \frac{1}{2}(B+S)$, where $B=S=0$. The only mesons with $B=S=0$ and $I=1$ are pions. Therefore, in order to conserve isotopic spin, the electron is assigned $I_3 = -1$.

The electron is chosen to be an isovector particle in isospin space and a fermion in physical space. There should be no difficulty with statistics, since we are dealing in two different spaces. The introduction of a "strong isospin" $I_3 = -1$ for the electron is analogous to Weinberg's introduction of a "weak isospin" $I_3 = -\frac{1}{2}$ for the electron in $SU(2) \times U(1)$ (Weinberg, 1967, 1971). Later, I will discuss how the model can incorporate gluons to derive a color $SU(3)_c$ theory.

First, consider the Yang-Mills and Higgs fields with a Lagrangian density (Yang and Mills, 1954; 't Hooft, 1971a):

$$\mathscr{L} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \frac{1}{2} D_\mu Q_a D_\mu Q_a \tag{1}$$

Table 1. Quantum Numbers Q/e and I_3 Assigned for Isovector ($I=1$) Particles in Our Model of the Strong Interaction[a]

Quantum number		
Q/e	I_3	Particle
+1	+1	π^+, e^+, W^+
−1	−1	π^-, e^-, W^-
0	0	G, γ, ν_e

[a]Although $W^{\pm}$ is involved in weak interactions, quantum numbers are given for it because our $SU(3)$ coupling converges with $SU(2) \times U(1)$ at the mass of the vector boson $W^{\pm}$. Here $G =$ gluon.

where

$$G^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + \frac{1}{2} e\varepsilon_{abc} W^b_\mu W^c_\nu \tag{2}$$

and the covariant derivative is

$$D_\mu Q_a = \partial_\mu Q_a + \frac{1}{2} e\varepsilon_{abc} W^b_\mu Q_c \tag{3}$$

W^a_μ is a triplet of isovector fields ($\mu = 0, \ldots, 3$ spacetime indices; $a = 1,2,3$ isospace indices), and the Higgs field Q_a is a triplet of isovector fields (Ryder, 1985). The Lagrangian density in equation (1) has a massless Higgs particle $M_H = 0$; however, there is a symmetry-breaking mechanism due to the Higgs field Q_a (Higgs, 1966; Kibble, 1967). For the electron, the tensor $G^a_{\mu\nu}$ and the covariant derivative apply to a spin-$\frac{1}{2}$ field in physical space and to an isospin-1 field in isospin space (Goddard and Olive, 1978). Since we are dealing with a spin-$\frac{1}{2}$ field in physical space, the Dirac quantization condition is satisfied (Dirac, 1931):

$$eg = \tfrac{1}{2} n\hbar c, \qquad n = 0, \pm 1, \ldots \tag{4}$$

where $g = (137/2)e$ is the magnetic charge of the magnetic monopole.

We consider first the electrons as shown in Figure 1. The Q_a receives a nonzero expectation value:

$$\langle Q_a \rangle^2 = F^2 \tag{5}$$

and the electron mass is given by the coupling

$$m_e = eF \tag{6}$$

The Lagrangian in equation (1) gives a particle mass spectrum with $\pi^\pm$ mesons as the exchange particles of the strong interaction in Figure 1. The solution of the field equations is assumed to be time-independent and spherically symmetric ('t Hooft, 1974). The Lagrangian in equation (1) is transformed by introducing the vector

$$r_a = (x,y,z), \qquad r_a^2 = r^2$$

and writing

$$Q_a(\mathbf{x},t) = r_a Q(r), \qquad W^a_\mu(\mathbf{x},t) = \varepsilon_{\mu ab} r_b W(r) \tag{7}$$

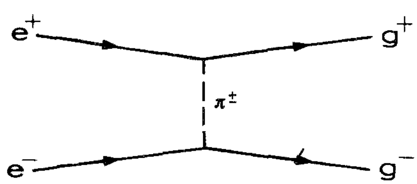

Fig. 1. $\pi^\pm$ mesons are the exchange particles of the strong interaction for the creation of magnetic monopoles.

where $\varepsilon_{\mu ab}$ are the structure constants of the gauge group, with $\varepsilon_{0ab}=0$ and $a=1,2,3$. The Lagrangian becomes

$$L=\int \mathcal{L}\,d^3\mathbf{x}=4\pi\int_0^\infty r^2\,dr\left[-r^2\left(\frac{dW}{dr}\right)^2-4rW\frac{dW}{dr}-6W^2-er^2W^3\right.$$
$$\left.-\frac{1}{8}e^2r^4W^4-\frac{1}{2}r^2\left(\frac{dQ}{dr}\right)^2-rQ\frac{dQ}{dr}-\frac{3}{2}Q^2-er^2WQ^2-\frac{1}{4}e^2r^4W^2Q^2\right] \tag{8}$$

't Hooft has studied the boundary condition at $r\to\infty$ for $Q(r)$ and $W(r)$. He finds for the fields ('t Hooft, 1974)

$$W_\mu^a(\mathbf{x},t)=-(\varepsilon_{\mu ab}/er^2)r_b,\qquad W(r)=-1/er^2$$

and

$$Q_a(\mathbf{x},t)=(F/r)r_a,\qquad Q(r)=F/r$$

The mass of the $\pi^\pm$ mesons is given by $E=-L$. Introducing the dimensionless parameters

$$w=2W(r)/F^2e,\qquad q=2Q/F^2e,\qquad x=\tfrac{1}{2}eFr$$

we obtain for the mass of the $\pi^\pm$ mesons (in rationalized units)

$$M_{\pi^\pm}=-L=\frac{2m_e}{e^2}\int_0^\infty x^2\,dx\left[x^2\left(\frac{dw}{dx}\right)^2+4xw\frac{dw}{dx}+6w^2+x^2w^3+\frac{1}{8}x^4w^4\right.$$
$$\left.+\tfrac{1}{2}x^2\left(\frac{dq}{dx}\right)^2+xq\frac{dq}{dx}+\frac{3}{2}q^2+x^2wq^2+\tfrac{1}{4}x^4w^2q^2\right] \tag{9}$$

The solution of this integral has been done by Prasad and Sommerfield (1975). The integral is evaluated to be 1.0 for a massless Higgs particle $M_{\mathrm{H}}=0$. Therefore, the mass of the $\pi^\pm$ mesons is

$$M_{\pi^\pm}=2m_e/e^2=137(2m_e)=140.0\ \mathrm{MeV} \tag{10}$$

The solution in equation (10) is within 0.31% of the experimental value: $M_{\pi^\pm}=139.57$ MeV (Particle Data Group, 1986). The result of equation (10) suggests that a renormalization of the Lagrangian might improve the calculations ('t Hooft, 1971b). Renormalization could include other exchange particles (i.e., K mesons, gluons, dyons, etc.) in the strong interaction. Introduction of gluons with $a=1,\ldots,8$ in the field tensor $G_{\mu\nu}^a$ would allow for a color $SU(3)_c$ theory of strong interactions (Ryder, 1985; 't Hooft, 1976).

3. MAGNETIC MONOPOLES AS ISOVECTOR PARTICLES

In this section, magnetic monopoles are treated as isovector particles in isospin space as shown in Table II. In order to derive the mass of the $\pi^\pm$ mesons, Maxwell's equations become

$$\partial_\mu F_{\mu\nu}=j_\nu,\qquad \partial_\mu\tilde{F}_{\mu\nu}=g_\nu \tag{11}$$

Table II. Quantum numbers M/g and I_3 Assigned to Magnetic Monopoles $g^{\pm}$ [a]

Quantum number		
M/g	I_3	Particle
+1	+1	g^+
−1	−1	g^-
0	0	G, γ, ν_g

[a] M/g is analogous to Q/e. Here ν_g is a magnetic-monopole type neutrino, which is predicted to exist by Lochak (1985). G = gluon.

where $\tilde{F}_{\mu\nu}$ is the dual tensor field, j_ν represents the sources of electric charges, and g_ν represents the sources of magnetic charges. $\tilde{F}_{\mu\nu}$ replaces $G^a_{\mu\nu}$ in the Lagrangian in equation (1). The magnetic monopole has a mass given by the coupling

$$M_g = gF \tag{12}$$

and by using the dual symmetry (Montonen and Olive, 1977)

$$e \to g; \qquad g \to -e \tag{13}$$

The field tensor (2) becomes

$$\tilde{F}^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + \tfrac{1}{2} g \varepsilon_{abc} W^b_\mu W^c_\nu \tag{14}$$

and the covariant derivative becomes

$$D_\mu Q_a = \partial_\mu Q_a + \tfrac{1}{2} g \varepsilon_{abc} W^b_\mu Q_c \tag{15}$$

By substituting equations (14) and (15) into the Lagrangian (1), we can follow the same mathematical treatment as in the last section and derive the mass of the $\pi^\pm$ mesons as

$$M_{\pi^\pm} = (1/g^2)(2M_g) \tag{16}$$

For a coupling constant $g^2/\hbar c = 137/4$ and a magnetic monopole mass $M_g = 2397$ MeV from the experimental evidence (Akers, 1986), we have

$$M_{\pi^\pm} = (4/137)(2M_g) = 140.0 \text{ MeV} \tag{17}$$

This is the same result as in equation (10) and is to be expected since the $\pi^\pm$ mesons are the exchange particles in this model. Equations (10) and (16) suggest that the magnetic monopole mass is related to the electron mass by

$$\frac{1}{g^2}(2M_g) = \frac{1}{e^2}(2m_e)$$

or

$$M_g = \frac{g^2}{e^2} m_e = \left(\frac{137}{2}\right)^2 m_e \tag{18}$$

which is the result found through different means by Amaldi (1968).

4. UNIFICATION OF THE STRONG, ELECTROMAGNETIC, AND WEAK COUPLINGS

In efforts to unify the strong, weak, and electromagnetic forces, Georgi and Glashow have suggested that a true unification would involve only one coupling strength, the fine-structure constant (Georgi and Glashow, 1974). In this section, the fundamental couplings of the strong and weak forces will be derived in terms of $\alpha = 1/137$. Furthermore, we will show why the masses of the intermediate vector boson W and the electron are so disparate $(M_W/m_e = 1.6 \times 10^5)$ as pondered by Llewellyn Smith (1983).

We start with the gauge theory of 't Hooft, in which the intermediate vector bosons $W^{\pm}$ are chosen as isovector particles and in which magnetic monopoles are admitted as solutions of the Lagrangian ('t Hooft, 1974). However, we use the dual field tensor (14) for spin-1 bosons:

$$\tilde{F}^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\varepsilon_{abc} W^b_\mu W^c_\nu \tag{19}$$

and a covariant derivative

$$D_\mu Q_a = \partial_\mu Q_a + g\varepsilon_{abc} W^b_\mu Q_c \tag{20}$$

The mass of the vector boson $W^{\pm}$ becomes

$$M_{W^{\pm}} = gF \tag{21}$$

Introducing the results (19) and (20) into the Lagrangian (1), we follow again the same mathematical treatment as in Section 2 and derive the mass of the magnetic monopole as

$$M_g = (1/g^2) M_W \tag{22}$$

for the $SU(3)$ magnetic monopoles in our model (Weinberg, 1973; Gross and Wilczek, 1973).

From a coupling constant $g^2/\hbar c = 137/4$ and a magnetic monopole mass $M_g = 2397$ MeV, we have

$$M_{W^{\pm}} = (137/4) M_g = 82.1 \text{ GeV} \tag{23}$$

This result is within 0.36% of the experimental value $M_{W^\pm} = 81.8 \pm 1.5$ GeV (Particle Data Group, 1986). Actually, the result (23) is within experimental uncertainty: $82.1 - 81.8 = 0.3$ GeV.

We can introduce the result of (18) into (22) and obtain

$$M_{W^\pm} = (g^4/e^2)m_e = \tfrac{1}{2}(137/2)^3 m_e \tag{24}$$

The mass of the intermediate vector boson W is given in terms of the fine-structure constant $\alpha = 1/137$ and the electron's mass m_e—two fundamental constants. From (24), we have

$$M_W/m_e = \tfrac{1}{2}(137/2)^3 = 1.6 \times 10^5$$

which is the result pondered by Llewellyn Smith (1983).

To calculate the weak coupling constant G_W in terms of the fine-structure constant, we turn to the electroweak model by Weinberg (1967, 1971). The mass of the vector boson W in Weinberg's model is

$$M_W = \tfrac{1}{2}m_e g'/G_e \tag{25}$$

where the e-ϕ coupling constant $G_e = 2.07 \times 10^{-6}$ and $G_e = 2^{1/4} m_e\ G_W^{1/2}$ (Weinberg, 1967). Combining (24) with (25), we obtain

$$g' = \left(\frac{1}{2\alpha}\right)^3 G_e = 0.665$$

or

$$G_e = 0.665(2\alpha)^3$$

The weak coupling constant is

$$G_W = \frac{G_e^2}{2^{1/2} m_e^2} = \frac{(0.665)^2(2\alpha)^6}{2^{1/2} m_e^2}$$

We can see that the coupling charge $g' = \tfrac{2}{3}e$, and the weak coupling constant is then

$$G_W = \frac{(2/3)^2(2\alpha)^6}{2^{1/2} m_e^2} = 1.165 \times 10^{-5}\ \text{GeV}^{-2}$$

The accepted value is $G_W = 1.16637 \times 10^{-5}\ \text{GeV}^{-2}$ (Particle Data Group, 1986). Thus, we have derived the weak coupling constant G_W in terms of α.

Finally, the strong coupling constant α_s can be derived from the results of magnetostrong theory by Akers (1986). The strong coupling constant was found to agree with experimental data when

$$\alpha_s = 2M_g/E \tag{26}$$

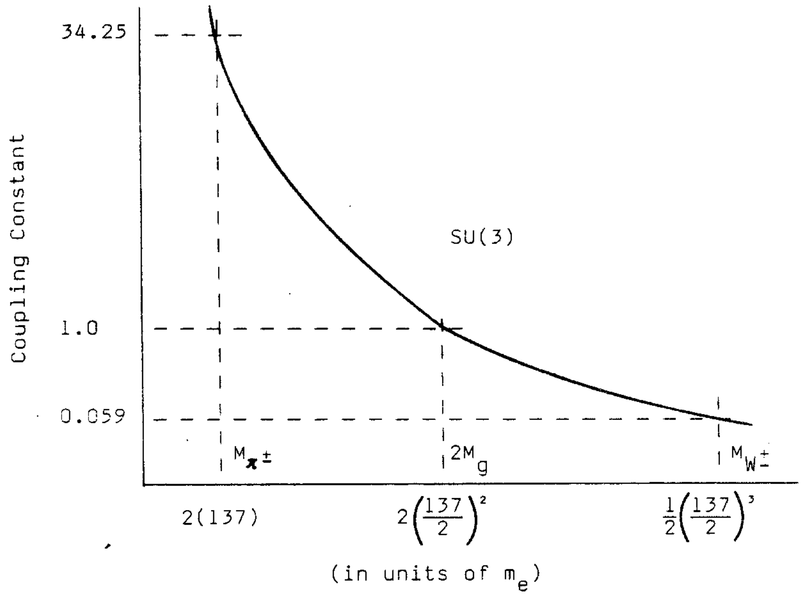

Fig. 2. The coupling constant as a function of the center-of-mass energy (in units of m_e). The $SU(3)$ model of this paper allows for $\pi^\pm$ mesons as the exchange particles of the strong interaction and for $W^\pm$ bosons in the weak interaction. The coupling constant converges at the mass of the intermediate vector boson $M_{W^\pm}$.

where E is the center-of-mass energy. $M_g = 2397$ MeV is the magnetic monopole mass. For a strong coupling constant on the order of unity,

$$E = 2M_g = 2(137/2)^2 m_e$$

or

$$E = [2/(2\alpha)^2]m_e \tag{27}$$

Thus,

$$\alpha_s = (2\alpha)^2 M_g/m_e \tag{28}$$

The idea that the strong forces of nature are due to magnetic charge was first suggested by Schwinger (1969) and later by Barut (1971a).

5. CONCLUSION

We summarize the results for particle masses in this paper:

$$M_{\pi^\pm} = 137(2m_e) = 2\alpha^{-1}m_e$$
$$M_g = (137/2)^2 m_e = (2\alpha)^{-2} m_e$$
$$M_{W^\pm} = \tfrac{1}{2}(137/2)^3 m_e = \tfrac{1}{2}(2\alpha)^{-3} m_e$$

$\pi^{\pm}$ mesons, magnetic monopoles, and vector bosons $W^{\pm}$ are fundamentally based upon the fine-structure constant α and the electron's mass m_e—two fundamental constants. The coupling constant in our $SU(3)$ model is shown in Figure 2. The strength of the coupling constant between magnetic monopoles is large:

$$\alpha_g = g^2/\hbar c = 34.25$$

As shown in Figure 2, this is the strength of our $SU(3)$ model with a center-of-mass energy equal to the mass of the $\pi^{\pm}$ mesons. The strong $SU(3)$ coupling converges with $SU(2) \times U(1)$ at the mass of the intermediate vector boson $W^{\pm}$, which agrees with the earlier results of magnetostrong theory by Akers (1986).

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